VALLADOLID LECTURE 3 EXERCISE

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Consider $f = x^2+5x^3y-y^3+1 \in \mathbb{C}[x^{\pm 1}, y^{\pm 1}]$, and $C = V(f) \subseteq (\mathbb{C}^*)^2$.

(1) Compute $\mathrm{trop}(C) \subseteq \mathbb{R}^2$, including all the multiplicities. Verify that these multiplicities make $\mathrm{trop}(C)$ balanced.

(2) Consider $\phi^* = \phi_A^*$, where

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

. Compute $g = \phi^{*-1}(f)$, and $\mathrm{trop}(V(g))$. Is this what you expect?

(3) Let $X = V(x^2+5x^3y-y^3+1, x+y+z-1) \subseteq (\mathbb{C}^*)^3$. Compute projections of X to two dimensional tori (for example, the x, y-torus, or the x, z torus. Can you get a tropical basis for X? (This might be hard). Your final tropical variety should be a one-dimensional fan with 6 rays.

The following `Macaulay2` code may help:

```
R=QQ[x,y,z];
I=ideal(x^2+5*x^3*y-y^3+1,x+y+z-1);
needsPackage "Tropical";
viewHelp Tropical;
X=tropicalVariety I;
rays X
eliminate(x,I)  -- computes the projection to the yz plane
```

You can use Macaulay2 on the web here: `https://www.unimelb-macaulay2.cloud.edu.au/#home` (google Macaulay2 TryItOut).