INTERNATIONAL SCHOOL ON TROPICAL GEOMETRY
TROPICAL IDEALS

EXERCISE SHEET 1

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Exercise 1. Consider the linear subspace L of $\mathbb{C}^5$ defined as

$$L = \text{rowspace} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 2 \end{pmatrix}.$$

(1) Compute generators for the ideal $I(L) \subset \mathbb{C}[x_1, \ldots, x_5]$ of all polynomial relations satisfied by L.
(2) Compute the matroid $M(L)$ on the ground set $\{1, \ldots, 5\}$, describing it in terms of circuits and bases.
(3) Is $M(L)$ a graphical matroid?
(4) Draw the lattice of flats of $M(L)$.
(5) Write down a collection of tropical equations that describe the polyhedral complex $\text{trop}(L)$ in tropical projective space $\mathbb{R}^5 / \mathbb{R} \cdot \mathbf{1}$ (i.e., write down a tropical basis).
(6) What is the topology of $\text{trop}(L) \cap S^3$?

Additional exercises:

Exercise 2. Let (V, E) be a graph. Show that the collection of subsets $C \subset E$ that form a non-self-intersecting cycle satisfy the circuit axioms of a matroid on the ground set E.

Exercise 3. Prove directly from the circuit axioms that all the bases of a matroid have the same cardinality.